

Elementary Mathematics Teaching Competencies

Standards for Prospective Elementary Mathematics Teachers

Part I

Mathematics Content Knowledge

Ways of thinking, understanding, communicating, and using mathematics, as described in the *Habits of Mind* section of this document, will enable future teachers to develop *Mathematics Pedagogical Knowledge*. It is therefore assumed that the *Mathematics Content Knowledge* will be demonstrated within a framework of those *Habits of Mind*. Furthermore, candidates will demonstrate knowledge of the historical and cultural development of each of the *Mathematics Content Knowledge* areas.

a. Number and Operations:

- Candidates understand and use a large repertoire of interpretations of addition, subtraction, multiplication and division, and of ways they can be applied.
- Candidates consistently use the concepts of place value in constructing efficient and varied representations of whole numbers and finite decimals, comparing and ordering numbers, and understanding the relative magnitude of numbers.
- Candidates understand multidigit calculations, including standard algorithms, “mental math,” and non-standard methods commonly created by students: the reasoning behind the procedures, how the base-10 structure of number is used in these calculations.
- Candidates understand concepts of integers and rationals: what integers and rationals (represented as fractions and decimals) are; a sense of their relative size; how operations on whole numbers extend to integers and rational numbers; and the behavior of units under the operations.

b. Algebra and Functions:

- Candidates explore and analyze patterns and functions, and describe, extend, create, and make generalizations about geometric and numeric patterns.
- Candidates apply commutativity, associativity, distributivity, identities, and inverses as properties of operations on a given domain; seeing computation algorithms as applications of particular axioms; appreciating that a small set of rules governs all of arithmetic.
- Candidates employ concepts of equality and relational thinking to solve problems, and utilize algebraic notation to represent calculation.
- Candidates understand different forms of argument and devise deductive arguments.
- Candidates use verbal, pictorial, tabular, symbolic, and graphic representations to emphasize relationships among quantities.

- c. Geometry:
- Candidates utilize visualization skills: are familiar with projections, cross-sections, and decompositions of common two- and three-dimensional shapes; represent three-dimensional objects in two dimensions, and construct three-dimensional objects from two-dimensional representations.
 - Candidates identify basic shapes, their properties, and relationships among them.
 - Candidates describe and apply transformations (reflections, rotations, and translations), congruence, and similarity.
 - Candidates communicate geometric ideas: use appropriate mathematical terminology and understand the role of mathematical definition.
- d. Measurement:
- Candidates determine non-standard units of measure or benchmark points to estimate measures.
 - Candidates understand the process of measurement: understand the idea of a unit and the need to select a unit appropriate to the attribute being measured, know U.S. Customary and metric systems, understand that measurements are approximate and that different units affect precision, are able to compare units and convert measurements from one unit to another.
 - Candidates understand length, area, and volume: see rectangles as arrays of squares, rectangular solids as arrays of cubes; recognize the behavior of measure (length, area, and volume) under uniform dilations; devise area formulas for basic shapes; understand the independence of perimeter and area, of surface area and volume.
- e. Data Analysis, Probability and Statistics:
- Candidates design data investigations: understand the kinds of question that can be addressed by data, create data sets, move back and forth between the question (the purpose of the study) and its design.
 - Candidates describe data: understand shape, spread, and center; use different forms of representation; compare two sets of data.
 - Candidates draw conclusions: choose among representations and summary statistics to communicate conclusions, understand variability, understand the difficulties that arise in sampling and inference.
 - Candidates use concepts of probability: make judgments under conditions of uncertainty, measure likelihood, understand randomness, apply experimental and theoretical probabilities to predict outcomes and test predictions.
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Part II
Mathematics Habits of Mind

- a. Problem Solving:
- Candidates engage in mathematical inquiry through understanding a problem, exploring, conjecturing, experimenting, and justifying.

- b. Reasoning and Proof:
 - Candidates select and use various types of reasoning including categorizing based on numeric and geometric properties, and using Venn diagrams, set notation and operations.
 - Candidates develop and evaluate mathematical arguments (i.e., informal proofs), and the foundations on which arguments are built.
 - c. Communication:
 - Candidates organize and consolidate their mathematical thinking through communication.
 - Candidates communicate coherently and use the language of mathematics (symbols and terminology) to express ideas precisely.
 - Candidates analyze the mathematical thinking of others.
 - d. Representation:
 - Candidates use multiple forms of representation including concrete models, pictures, diagrams, tables, and graphs.
 - Candidates use invented and conventional terms and symbols to communicate reasoning and solve problems.
 - e. Connections:
 - Candidates understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
 - Candidates recognize and apply mathematics in contexts outside of mathematics.
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Part III
Mathematics Pedagogical Knowledge

- a. Equity:
 - Candidates demonstrate high expectations and strong support for all students to learn mathematics.
- b. Curriculum:
 - Candidates map curriculum that is coherent, focused on important mathematics, and carefully sequenced.
 - Candidates are familiar with curriculum both preceding and following their teaching level.
 - Candidates are able to discern the quality of learning opportunities for students when given a particular task, activity, educational software, etc. and are able to make adaptations to assure quality.

c. Learning Environment:

- Candidates foster a classroom environment conducive to mathematical learning through
 - providing and structuring the time necessary to explore sound mathematics and grapple with significant ideas and problems;
 - using the physical space and materials in ways that facilitate students' learning of mathematics;
 - providing a context that encourages the development of mathematical skill and proficiency;
 - respecting and valuing students' ideas, ways of thinking, and mathematical dispositions.

d. Teaching:

- Candidates understand what mathematics students know and need to learn and then challenge and support them to learn it well.
- Candidates orchestrate discourse by
 - posing questions and tasks that elicit, engage, and challenge each student's thinking;
 - listening carefully to students' ideas; asking students to clarify and justify their ideas orally and in writing;
 - deciding what to pursue in depth from among the ideas that students bring up during a discussion;
 - deciding when and how to attach mathematical notation and language to students' ideas; deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty;
 - monitoring students' participation in discussions and deciding when and how to encourage each student to participate.

e. Learning:

- Candidates know that students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.
- Candidates have the ability to recognize and move students from concrete to abstract levels of understanding.

f. Assessment:

- Candidates use a variety of formal and informal, formative and summative assessment techniques to support the learning of important mathematics.
- Candidates understand how, why, and when to use various assessment techniques and tools – as well as how these inform their understanding about student thinking and understanding.
- Candidates plan instruction based upon the information obtained through classroom and external assessments of each student's developmental level.

- g. Technology:
- Candidates understand that technology is an integral part of teaching and learning mathematics both influencing what is taught and enhancing how it is learned.
 - Candidates demonstrate effective and appropriate use of technology.
- h. Historical Development:
- Candidates demonstrate knowledge of historical and cultural influences in mathematics including contributions of underrepresented groups and incorporate this knowledge into instruction.
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