Basic Mathematics Teaching Competencies

Standards for Prospective Middle School Mathematics Teachers

Part I

Mathematics Content Knowledge

Ways of thinking, understanding, communicating, and using mathematics, as described in the *Habits of Mind* section of this document, will enable future teachers to develop *Mathematics Pedagogical Knowledge*. It is therefore assumed that the *Mathematics Content Knowledge* will be demonstrated within a framework of those *Habits of Mind*. Furthermore, candidates will demonstrate knowledge of the historical and cultural development of each of the *Mathematics Content Knowledge* areas.

Proportionality is an integrative theme in the middle level mathematics program. Facility with proportionality develops through work in many areas of the curriculum, including ratio and proportion, percent, similarity, scaling, linear equations, slope, relative-frequency histograms, and probability. Other themes in middle level mathematics are those of variables and relations, as fostering algebraic thinking at the middle level is critical to further work in mathematics.

a. Number and Operations:
   - Candidates demonstrate conceptual understanding of real (particularly rationals and integers) and complex numbers; ways of representing number; relationships among number and number systems; and the meaning of operations.
   - Candidates are computationally proficient and choose the appropriate computational format (exact or approximate) and method (mental, paper and pencil, or electronic).

b. Algebra and Functions:
   - Candidates understand the various roles of algebra and demonstrate conceptual understanding of variables and functions including linear, quadratic, and exponential functions and their inverses.
   - Candidates use a variety of representations including verbal, pictoral, tabular, symbolic and graphic to emphasize relationships among quantities.
   - Candidates demonstrate conceptual understanding of and skill in appropriate use of symbols.

c. Geometry:
   - Candidates use spatial visualization and geometric modeling and constructions to explore and analyze geometric shapes, structures, and their properties.
   - Candidates make conjectures about two- and three-dimensional shapes and offer justifications for conjectures.
   - Candidates apply coordinate geometry and transformations including the use of congruence, similarity, and symmetry to analyze mathematical situations.

d. Measurement:
   - Candidates understand measurement processes including estimation, accuracy, and choice of measurement tool for both U.S. Customary and metric systems.
   - Candidates understand and use direct and indirect measurement techniques and formulas for both two- and three-dimensional figures.

e. Data Analysis, Probability and Statistics:
   - Candidates design investigations, collect data, use a variety of ways to display the data and critically interpret data representations.
   - Candidates make predictions and draw conclusions involving uncertainty by applying basic concepts of probability.
   - Candidates use appropriate statistical methods to analyze and describe shape, spread, and center data; then they use that information to make inferences.

f. Calculus:
   - Candidates demonstrate a conceptual understanding of limit, particularly in relation to understanding series, repetitive processes, and non-terminating decimals.
   - Candidates demonstrate a conceptual understanding of rate of change and the relationship to minimums, maximums, and area of a region.

Part II
Mathematics Habits of Mind

a. Problem Solving:
   - Candidates engage in mathematical inquiry through understanding a problem, exploring, conjecturing, experimenting, and justifying.

b. Reasoning and Proof:
   - Candidates select and use various types of reasoning including categorizing based on numeric and geometric properties, and using Venn diagrams, set notation and operations.
   - Candidates develop and evaluate mathematical arguments (i.e., informal proofs), and the foundations on which arguments are built.

c. Communication:
   - Candidates organize and consolidate their mathematical thinking through communication.
   - Candidates communicate coherently and use the language of mathematics (symbols and terminology) to express ideas precisely.
   - Candidates analyze the mathematical thinking of others.

d. Representation:
   - Candidates use multiple forms of representation including concrete models, pictures, diagrams, tables, and graphs.
   - Candidates use invented and conventional terms and symbols to communicate reasoning and solve problems.

e. Connections:
   - Candidates understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
   - Candidates recognize and apply mathematics in contexts outside of mathematics.

Part III
Mathematics Pedagogical Knowledge

a. Equity:
   - Candidates demonstrate high expectations and strong support for all students to learn mathematics.

b. Curriculum:
   - Candidates map curriculum that is coherent, focused on important mathematics, and carefully sequenced.
   - Candidates are familiar with curriculum both preceding and following the middle level.
   - Candidates are able to discern the quality of learning opportunities for students when given a particular task, activity, educational software, etc. and are able to make adaptations to assure quality.

c. Learning Environment:
   - Candidates foster a classroom environment conducive to mathematical learning through
     - providing and structuring the time necessary to explore sound mathematics and grapple with significant ideas and problems;
     - using the physical space and materials in ways that facilitate students' learning of mathematics;
     - providing a context that encourages the development of mathematical skill and proficiency;
     - respecting and valuing students' ideas, ways of thinking, and mathematical dispositions.

d. Teaching:
   - Candidates understand what mathematics students know and need to learn and then challenge and support them to learn it well.
   - Candidates orchestrate discourse by
     - posing questions and tasks that elicit, engage, and challenge each student's thinking;

- listening carefully to students' ideas; asking students to clarify and justify their ideas orally and in writing;
- deciding what to pursue in depth from among the ideas that students bring up during a discussion;
- deciding when and how to attach mathematical notation and language to students' ideas; deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty;
- monitoring students' participation in discussions and deciding when and how to encourage each student to participate.

e. Learning:
   - Candidates know that students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.
   - Candidates have the ability to recognize and move students from concrete to abstract levels of understanding.

f. Assessment:
   - Candidates use a variety of formal and informal, formative and summative assessment techniques to support the learning of important mathematics.
   - Candidates understand how, why, and when to use various assessment techniques and tools – as well as how these inform their understanding about student thinking and understanding.
   - Candidates plan instruction based upon the information obtained through classroom and external assessments of each student's developmental level.

g. Technology:
   - Candidates understand that technology is an integral part of teaching and learning mathematics both influencing what is taught and enhancing how it is learned.
   - Candidates demonstrate effective and appropriate use of technology.

h. Historical Development:
   - Candidates demonstrate knowledge of historical and cultural influences in mathematics including contributions of underrepresented groups.

Bibliography

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