Advanced Mathematics Teaching Competencies

Standards for Prospective Secondary Mathematics Teachers

Part I

Mathematics Content Knowledge

Ways of thinking, understanding, communicating, and using mathematics, as described in the *Habits of Mind* section of this document, will enable future teachers to develop *Mathematics Pedagogical Knowledge*. It is therefore assumed that the *Mathematics Content Knowledge* will be demonstrated within a framework of those *Habits of Mind*. Furthermore, candidates will demonstrate knowledge of the historical and cultural development of each of the *Mathematics Content Knowledge* areas.

a. Number, Operations, and Algebra:

- Candidates demonstrate knowledge of the properties of the natural, integer, rational, real, and complex number systems and the interrelationships of these number systems.
- Candidates identify and apply the basic ideas, properties and results of number theory and algebraic structures that underlie numbers and algebraic expressions, operations, equations, and inequalities.
- Candidates use algebraic equations to describe lines, planes and conic sections and to find distances in the plane and space.
- Candidates demonstrate the use of algebra to model, analyze, and solve problems from various areas of mathematics, science, and the social sciences.
- Candidates apply properties and operations of matrices and techniques of analytic geometry to analyze and solve systems of equations.
- Candidates use graphing calculators, computer algebra systems, and spreadsheets to explore algebraic ideas and algebraic representations of information, and to solve problems.

b. Geometry:

- Candidates demonstrate knowledge of core concepts and principles of Euclidean geometry in the plane and space and some knowledge of geometries that are not Euclidean.
- Candidates describe properties, measures, and relationships of geometric figures such as various triangles, quadrilaterals, general polygons, and conic sections.
- Candidates employ a variety of methods and associated geometric concepts and representations, including transformations, coordinates, and vectors to analyze and solve problems.
- Candidates explore significant geometry topics and applications such as tiling, fractals, computer graphics, and robotics.
- Candidates use a variety of geometric and computer tools to conduct geometric investigations.

c. Functions:

- Candidates demonstrate knowledge of the concept of a function and the most important classes of functions, including polynomial, exponential and logarithmic, rational, and trigonometric.
- Candidates represent functions in multiple forms, such as graphs, tables, mappings, formulas, matrices, and equations.
- Candidates perform a variety of operations on functions, including addition, multiplication and composition of functions, and recognize related special functions such as identities and inverses and those operations that preserve the various properties.
- Candidates use functions to model situations and solve problems in calculus, linear and abstract algebra, geometry, statistics, and discrete mathematics.
- Candidates explore various kinds of relations, including equivalence relations, and the differences between relations and functions.
- Candidates use calculator and computer technology effectively to study functions and solve problems.
- Candidates demonstrate specific knowledge of trigonometric functions, including properties of their graphs, special angles, identities and inequalities, and complex and polar forms.
- Candidates use analytic representations, measures, and properties to analyze transformation of two- and three-dimensional objects.

d. Calculus:

- Candidates demonstrate conceptual understanding of and procedural facility with basic calculus concepts such as limits, derivatives, and integrals of functions of one and two variables.
- Candidates use concepts of calculus to analyze the behavior of functions and solve problems.
- Candidates determine the limits of sequences and series, and demonstrate the convergence or divergence of series.

e. Probability and Statistics:

- Candidates explore data using a variety of standard techniques to organize and display data and detect and use measures of central tendency and dispersion
- Candidates use surveys to estimate population characteristics and design experiments to test conjectured relationships among variables.
- Candidates use theory and simulations to study probability distributions and apply them as models of real phenomena.
- Candidates demonstrate knowledge of statistical inference by using probability models to draw conclusions from data and measure the uncertainty of those conclusions.
- Candidates employ calculators and computers effectively in statistical explorations and practice.
- Candidates demonstrate knowledge of basic concepts of probability such as conditional probability and independence, and develop skill in calculating probabilities associated with those concepts.

f. Discrete Mathematics and Computer Science:
- Candidates demonstrate knowledge of discrete topics including graphs, trees, and networks, enumerative combinatorics, and finite difference equations, iteration, and recursion, and the use of tools such as functions, diagrams, and matrices to explore them.
- Candidates build discrete mathematical models for social decision-making.
- Candidates apply discrete structures (sets, logic, relations, and functions) and their applications in design of data structures and programming.
- Candidates use recursion and combinatorics in the design and analysis of algorithms
- Candidates employ linear and computer programming to solve problems.

Part II
Mathematics Habits of Mind

a. Problem Solving:
- Candidates engage in mathematical inquiry through understanding a problem, exploring, recognizing patterns, conjecturing, experimenting, and justifying.

b. Reasoning and Proof:
- Candidates select and use various types of reasoning and develop and evaluate mathematical arguments and proof in all the mathematics content knowledge areas.

c. Communication:
- Candidates organize and consolidate their mathematical thinking through communication.
- Candidates communicate coherently and use the language of mathematics (symbols and terminology) to express ideas precisely.
- Candidates analyze the mathematical thinking of others.

d. Representation:
- Candidates use multiple forms of representation including concrete models, pictures, diagrams, tables, and graphs.
- Candidates use invented and conventional terms and symbols to communicate reasoning and solve problems.

e. Connections:
- Candidates understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Candidates recognize and apply mathematics in contexts outside of mathematics.

Part III

Mathematics Pedagogical Knowledge

a. Equity:
   - Candidates demonstrate high expectations and strong support for all students to learn mathematics.

b. Curriculum:
   - Candidates map curriculum that is coherent, focused on important mathematics, and carefully sequenced.
   - Candidates are familiar with curriculum both preceding and following the high school level.
   - Candidates are able to discern the quality of learning opportunities for students when given a particular task, activity, educational software, etc. and are able to make adaptations to assure quality.

c. Learning Environment:
   - Candidates foster a classroom environment conducive to mathematical learning through
     - providing and structuring the time necessary to explore sound mathematics and grapple with significant ideas and problems;
     - using the physical space and materials in ways that facilitate students' learning of mathematics;
     - providing a context that encourages the development of mathematical skill and proficiency;
     - respecting and valuing students' ideas, ways of thinking, and mathematical dispositions.

d. Teaching:
   - Candidates understand what mathematics students know and need to learn and then challenge and support them to learn it well.
   - Candidates orchestrate discourse by
     - posing questions and tasks that elicit, engage, and challenge each student's thinking;
     - listening carefully to students' ideas; asking students to clarify and justify their ideas orally and in writing;
     - deciding what to pursue in depth from among the ideas that students bring up during a discussion;
     - deciding when and how to attach mathematical notation and language to students' ideas; deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty;
     - monitoring students' participation in discussions and deciding when and how to encourage each student to participate.

e. Learning:
   - Candidates know that students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.
   - Candidates have the ability to recognize and move students from concrete to abstract levels of understanding.

f. Assessment:
   - Candidates use a variety of formal and informal, formative and summative assessment techniques to support the learning of important mathematics.
   - Candidates understand how, why, and when to use various assessment techniques and tools – as well as how these inform their understanding about student thinking and understanding.
   - Candidates plan instruction based upon the information obtained through classroom and external assessments of each student's developmental level.

g. Technology:
   - Candidates understand that technology is an integral part of teaching and learning mathematics both influencing what is taught and enhancing how it is learned.
   - Candidates demonstrate effective and appropriate use of technology.

h. Historical Development:
   - Candidates demonstrate knowledge of historical and cultural influences in mathematics including contributions of underrepresented groups.

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